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Heuts, R.M.J.; Strijbosch, L.W.G.; van der Schoot, E.H.M.

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A combined forecast-inventory control procedure for spare parts

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R.M.J. Heuts[1], L.W.G. Strijbosch[2] and E.H.M. van der Schoot[3]

[1] CentER for Economic Research, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, The Netherlands
[2] Faculty of Economics and Business Administration, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, The Netherlands
[3] CMG Trade, Transport & Industry, SAP Competence Center, P.O. Box 8566, 3009 AN, The Netherlands

Abstract

The interaction between forecast and inventory control has been under-exposed in the literature. This paper introduces a combined forecast-inventory control procedure that is appropriate especially for the lumpy demand character of spare parts. The proposed procedure is succesfully implemented in practice.

1. Introduction

MARS (at Veghel, The Netherlands), produces confectionery for the European market with highly automated machines, 24 hours per day, 365 days per year. A spare parts distribution centre accommodates all necessary spare parts in order to maintain the equipment. This centre is divided into two sections: an open (for 'cheep' parts) and a closed (for 'expensive' and/or very critical parts; often with low usage) storage section. The idea behind the origin of the open storage section is that the controlling costs of cheep spare parts are often much higher than the purchase price for such parts. The open storage section has been created in 1991 and yielded reduction of waiting times for repairmen and savings on total controlling costs.

In the open storage section the repairmen can retrieve cheep spare parts without making any registration. Two distributors nearby the distribution centre take care of the inventory control. Those distributors have to deliver spare parts within four working days, by contract. Therefore these distributors have to stock enough spare parts. In the open storage section the *physical stock* is controlled every day by a controller of the distributors. This controller uses a kind of (s, Q)-inventory control system, where the parameters are set manually and by experience, taking into account that ordering should take place in multiples of package sizes. When the physical inventory is beneath its order point then the code of this service part is scanned. The computer system SAP/R3 checks if already an order is outstanding for scanned parts. If not the part is ordered. The daily visual control of the physical inventory implies that no registrations of usage are needed. When a service part is not on stock, then it is almost always available at the nearby distributor. The delivery time is within four days more or less without variation.

In the closed storage section repairmen ought to register the retrieval of parts. MARS itself controls the inventory position in that case. In this section also a kind of (s,Q)-inventory control is used, whereby every transaction is registered. Again the reorder point and order size are based on intuition and experience regarding inventory turn over rates, product value and transportation/ordering costs. There is an endeaver to restrict the number of different distributors from 250 to 100. The lead-times from those distributors are quite different in length. Some variation in lead-time is possible; however, this mainly depends on the arrangements on deliverance. To reduce complexity we therefore assume fixed lead-times. After delivery of the service parts, they are checked and registered in the SAP/R3 system. The research project undertaken was primarily adressed to this closed section part.

Of all spare parts at MARS 40% had no usage during the years 1995-1997 and even 80% had no usage (cf. Table 1, Usage=0) in the period from Sept.'97 to Febr.'98. Only 1.2% spare parts had a mean usage per day of more than 0.15 (cf. Table 1, Usage>0.1). Thus, demand for spare parts is very erratic.

Table 1: *Relative frequency of daily usage during Sept.'97- Febr.'98*

Usage	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	2	3	4	>4.5
%	80	19	0.5	0.2	0.2	0.1	0.04	0.02	0.02	0.01	0.02	0.03	0.03	0.01	0.03

As the company is aware of the intuitive nature of the inventory control procedure, a study was undertaken (van der Schoot, 1998) to design and test a procedure which could outperform the existing one in service and/or costs. Management considers it important to have one well-defined procedure, which could be used for all spare parts, independent of their demand characteristics. Given the information lack regarding the performance of the existing policy and the intuitive nature of it, we firstly had to invent a simple and formal control rule approximating closely or even outperforming the existing one. In this way it would become possible to compare the simple approach adhered by the company with a more advanced policy involving more computer time, a less intuitive decision rule, and formulas which are difficult to comprehend by the employees concerned. Furthermore, with formally described inventory policies a simulation study could be set up to investigate the performance of both (the existing and the proposed) systems. It is clear that only substantial improvements upon the existing policy could convince management to pass on to a newly designed more complex system.

The next section discusses the characteristics of the spare parts demand and some possible control procedures. After introducing the necessary notation we formulate the used models precisely and describe the main results of a Monte Carlo investigation. The last section contains the conclusions and suggestions for further research.

2. Some remarks on spare part data and model selection

Ten representative spare parts were selected from the closed storage section to get an impression of the demand data of spare parts at MARS. For these ten parts the mean time (days) between usage (interarrival time), $E(A)$, the mean demand when the part is required, $E(D^*)$, and the demand variance, $Var(D^*)$ were obtained empirically. Together with the fixed lead-time (in days), L, the parts value, v, and the ordering cost, these figures are recorded in Table 2 illustrating the variability of the main characteristics which is representative for many parts. Some parts are needed seldom (M8, M9), while the required number is 1 or 2 (M9), or varies between 1 and for example 7 (M8); other parts are needed frequently like M0, M1, M2, M4 also with strongly varying required numbers. Part M2, a very critical part in spite of the low price, is measured in centimeters. The coefficient of variation of the interarrival time which is an important characteristic when designing a new control procedure, appears to be close to one as can be expected from the nature of demand (failures of machines). Due to the limited period for which historical information was available this could not be checked for M8 and M9, unfortunately. However, we assume from now on that this coefficient of variation equals $\sqrt{1-1/E(A)}$, the coefficient of variation of the underlying geometric variable.

Table 2: *Some data information on ten selected spare parts.*

Material	$E(A)$	$E(D^*)$	$Var(D^*)$	Lead-time (L)	Value (v)	Ordering cost
M0	5.9	1.6	1.0	5	14.50	13.62
M1	7.0	21.0	447.6	5	11.83	13.62
M2	7.5	122.7	7709.6	14	1.18	38.62
M3	7.7	3.4	14.2	14	28.00	38.62
M4	24.0	19.4	48.2	30	58.00	38.62
M5	24.5	2.8	4.4	21	22.50	38.62
M6	24.7	7.0	8.0	30	20.02	13.62
M7	24.8	1.2	0.2	25	93.65	38.62
M8	119.0	2.5	4.5	31	725.00	38.62
M9	204.0	1.5	0.5	42	444.00	38.62

Lead-time variability is sufficiently low (see introduction) to assume it to be deterministic. Table 2 demonstrates that demand is very intermittent, in general. A well-defined control policy has to account for these demand patterns. In consultation with the company a service criterion should be used to determine the decision parameters s and Q. The main reason for this choice is the lack of cost information on the various cost components needed for a cost model. Of the commonly used service criteria the so-called P_2-

criterion (also called the fill rate) has been chosen, meaning that on the average during a delivery cycle, a shortage of $100(1-P_2)$ percent of the mean demand during an arbitrary cycle is allowed.

In the literature several approaches (Silver *et al.*, 1998) are available for the determination of the parameters s and Q in an (s,Q)-inventory model,where s is the reorder point and Q the order quantity. Assuming normality of demand during lead-time, the reorder point s can be obtained from

(1) $s = \hat{x}_L + k\hat{\sigma}_L$

where the safety stock $k\hat{\sigma}_L$ is the product of the forecast error standard deviation of demand during the lead-time and the safety factor k, which very much depends on the desired type of service and the normality assumption. Some objections against the normality assumption are:

- non-unit demand can lead to an inventory level under s when the order is placed;
- a large coefficient of variation leads to an unacceptable large chance of negative demands in the model;
- skewness is not taken into account.

However, the simplicity of the reorder point formula and its relatively easy implementation explains its widely usage in many inventory management systems and commercial software. This standard procedure is a useful candidate to approach the existing intuitive control procedure of MARS. It is simple to understand, applicable in many situations and -what is most important- closely related to what management considers appropriate. As management is inclined to use equal safety factors over different spare parts, individually determined safety factors should outperform the system adhered by the company. Then, showing that an advanced system is even better only requiring the implementation of some software and a little more computer time, better values for the decision parameters could be determined on a regular basis, yielding a closer approximation of the desired service level.

For spare parts inventory management, the literature often suggests the compound Poisson process concept for demand during lead-time. However, using a compound Poisson process assumes that empirical information is available of each individual demand and its demand size. In practice this is often not a very realistic starting point. Rather one has information on discrete time units, such as days or weeks, where demand is aggregated over time units. Also in the present case one works with demand per day information. Janssen *et al.* (1998) describe an (R,s,Q) inventory model with a service level restriction, and where demand is modelled as a compound Bernoulli process, which very much resembles the compound Poisson modelling concept when the time unit is chosen small. They have shown that this kind of modelling is especially suitable for intermittent demand. Further they also show that incorporating the undershoot of the reorder level yields a substantial improvement on the attained performance levels when demand is intermittent. However, in the article of Janssen *et al.* (1998) this procedure is only tested with known parameter values of the demand distribution. The Bernoulli demand modelling concept seems a very good candidate for using it as the basis of a newly designed more advanced system for application in the present case.

Forecasting unknown demand parameters

A problem of many methods, suggested in the literature, is the limited reported experience of applying these methods in practice. It is often unknown to what extent an a priori chosen performance level is attained. The performance of these methods is not known when using parameter estimates in stead of the true values of the parameters. Silver and Rahnama (1987) reported on a (s,Q)-model with a cost criterion, where underestimating the safety factor tends to lead to a higher cost penalty than equivalently overestimating it. Therefore they suggest to bias the safety factor deliberately upwards so as to reduce the expected cost penalty associated with statistically estimating the reorder point. An important conclusion of this study should be that all methods, which are advocated in literature, should be tested, not with complete information on those parameters, but with an estimation procedure included. The common way to produce estimates of parameters of demand distributions is by forecasting. Especially exponential smoothing is applied, for its ability to incorporate non-stationary behaviour of demand. Strijbosch *et al.* (1997) and Strijbosch and Moors (1999) investigate the effect of exponentially smoothing demand analytically and by simulation in a (R,S)-control policy when demand is normally or gamma distributed and leadtimes are zero. Table 2 suggests that simple exponential smoothing is not appropriate. Croston (1972) showed that when demand is intermittent, which is obviously the case for spare parts, the forecast error can be reduced by smoothing the time between demands and the demand sizes separately, and using these for the forecasting of lead-time demand. Many authors after 1972 showed the relevance of this Croston method.

3. Notation

The notation to be used includes

L = the deterministic lead-time in days

D_t = total demand on day t

$Z_t(L) = \sum_{\tau=1}^{L} D_{t+\tau}$, total demand during the days $t+1,...,t+L$

$\hat{D}_t(\tau)$ = forecast of demand on day t for day $t+\tau$

$\hat{Z}_t(L) = \sum_{\tau=1}^{L} \hat{D}_t(\tau)$, forecast of demand during the days $t+1,...,t+L$

$e_t(\tau) = D_{t+\tau} - \hat{D}_t(\tau)$, forecast error for $D_{t+\tau}$

$E_t(L) = \sum_{\tau=1}^{L} e_t(\tau)$, forecast error for $Z_t(L)$

p = the probability of a positive demand on a day

A_z = the z-*th* interarrival time (in days) between two days with positive demand;

$$P(A_z = k) = (1-p)^{k-1} p, \quad k \geq 1; \; E(A_z) = 1/p; \; Var(A_z) = (1-p)/p^2$$

D_z^* = the z-*th* (total) demand size on a day (if positive) ; $E(D_z^*) = a$; $Var(D_z^*) = \sigma^2$

Q = fixed order quantity when the inventory position gets under the reorder point

s = the reorder point

$F_D(\cdot)$ = the distribution function of D_t

$F_{D^*}(\cdot)$ = the distribution function of D_z^*

$Z^*(L)$ = the demand during the lead-time if it is positive.

p_L = probability of positive demand during the lead-time

U = the undershoot of the reorder point

Z^* = $Z^*(L) + U$

c_{D^*} $= \sigma / a$, the coefficient of variation of D_z^*

$(x)^+$ = $\max(x, 0)$

4. Model formulation

This section will be split up into three subsections; one on the forecasting approach, the next on the simple inventory model which will be used to simulate the company's preferred model and the last on the advanced model which should outperform the simple model. Both models apply the forecasting method explained in the next subsection.

4.1 Forecasting the intermittent demand structure of the spare parts

In the (s,Q) inventory models described in the next two subsections it is of interest to calculate forecasts, not for a single period, but rather the sum of forecasts for the next L periods, $\hat{Z}_t(L)$. The forecast error is given by $E_t(L) = Z_t(L) - \hat{Z}_t(L) = \sum_{\tau=1}^{L} D_{t+\tau} - \sum_{\tau=1}^{L} \hat{D}_t(\tau)$. As we are dealing with spare parts, seasonal or trend effects are not very likely (except in some special cases). Thus a forecasting procedure based on single exponential smoothing (SES) seems to be a good choice. As a consequence, all $\hat{D}_t(\tau), \tau = 1..., L$ are identical. For safety stock determination, we are interested in

(2) $$Var\{E_t(L)\} = Var\left\{\sum_{\tau=1}^{L} D_{t+\tau}\right\} + Var\left\{\sum_{\tau=1}^{L} \hat{D}_t(\tau)\right\}$$

As we assume that all D_t are i.i.d., $\hat{D}_t(1) = \hat{D}_t(\tau), \tau = 2..., L$, and $D_{t+\tau}, \tau = 1,..., L$ are independent of $\hat{D}_t(1)$, the next expression follows:

(3) $$Var\{E_t(L)\} = L\ Var\{D_{t+\tau}\} + L^2\ Var\{\hat{D}_t(1)\}$$

One (or more) days of usage and a large number of days of non-usage typically characterize the demand of spare parts. Consequently, we consider D_t as the product of two independent variables *x* and *y*, where *x* has a Bernoulli distribution with parameter *p* ($x = 1$ means: demand is positive) and *y* having the same distribution as D_z^* with mean *a* and variance σ^2. For this type of intermittent (or lumpy) demand Croston (1972) introduced a point forecasting procedure, which separates interarrival times of non-usage and order sizes at a demand occurrence. Johnston and Boylan (1996) and Willemain *et al.* (1994) show that this idea of Croston is superior to applying SES to all the demand data, zero demand or not. We employ this idea by transforming the demand data D_t into the series D_z^* and A_z such that index *z* consecutively only numbers the days with positive (total) demand (denoted by D_z^*), while A_z is the number of days since the previous day of positive demand. On both D_z^* and A_z SES is applied. To be more precise,

(4) $$\hat{D}_z^*(1) = \alpha\, D_z^* + (1-\alpha)\hat{D}_{z-1}^*(1)$$

(5) $$\hat{A}_z(1) = \beta\, A_z + (1-\beta)\hat{A}_{z-1}(1)$$

(6) $$\hat{D}_t(\tau) = \frac{\hat{D}_z^*(1)}{\hat{A}_z(1)}, \quad \tau = 1,2,...$$

where indices *z* and *t* are related in an obvious way: D_z^* (having mean *a* and variance σ^2) is the last observed (positive) demand on day *t* or previous to day *t*. Using the approximations $E\{\hat{D}_z^*(1)\} \approx a$, $E\{\hat{A}_z(1)\} \approx 1/p$, $Var\{\hat{D}_z^*(1)\} \approx \frac{\alpha}{2-\alpha}\sigma^2$, $Var\{\hat{A}_z(1)\} \approx \frac{\beta}{2-\beta}\frac{1-p}{p^2}$ (these approximations hold when enough historical information is available and the demand data is stationary; see e.g. Brown, 1963), and $Var(x/y) \approx \mu_x^2/\mu_y^2(\sigma_x^2/\mu_x^2 + \sigma_y^2/\mu_y^2)$ (for independent variables *x* and *y*; see Mood *et al.*, 1974) we may write

(7) $$Var\{\hat{D}_t(1)\} \approx p^2\left(\frac{\alpha}{2-\alpha}\sigma^2 + \frac{\beta}{2-\beta}(1-p)a^2\right)$$

As D_t can be considered as the product of two independent variables *x* and *y* (see before), the next expression follows (using $Var(xy) = \mu_y^2\sigma_x^2 + \mu_x^2\sigma_y^2 + \sigma_x^2\sigma_y^2$ for independent variables *x* and *y*; see Mood *et al.*, 1974):

(8) $$Var\{D_{t+\tau}\} \approx p\sigma^2 + a^2 p(1-p)$$

As a result

(9) $$Var\{E_t(L)\} \approx pL\left\{pL\left(\frac{\alpha}{2-\alpha}\sigma^2 + \frac{\beta}{2-\beta}(1-p)a^2\right) + \sigma^2 + a^2(1-p)\right\}$$

In this expression the unknown parameters can be estimated as follows:

(10) $$\hat{p} = 1/\hat{A}_z(1), \quad \hat{a} = \hat{D}_z^*(1), \quad \hat{\sigma} = 1.25\, MAD_z \sqrt{(2-\alpha)/2}$$

yielding $V\hat{a}r(E_t(L))$ as an estimate of $Var(E_t(L))$. The estimate for σ is obtained according to a well-known relationship (Brown, 1963) and MAD_z is obtained by SES too:

(11) $$MAD_z = \omega \,|\, D_z^* - \hat{D}_{z-1}^* \,| + (1-\omega) MAD_{z-1}$$

4.2. The simple (s, Q) inventory model as an approximation for a MARS intuitive approach

When applying the simple method (1), we need the estimates $\hat{x}_L$ and $\hat{\sigma}_L$. These can be obtained with the aforementioned forecasting procedure:

(12) $$\hat{x}_L = \hat{Z}_t(L)$$

(13) $$\hat{\sigma}_L^2 = V\hat{a}r(E_t(L))$$

The safety factor *k* can be obtained by solving the corresponding service equation (Silver *et al.*, 1998):

$G_u(k) = \dfrac{Q}{\sigma_L}(1 - P_2)$ where $G_u(k)$ is defined $G_u(k) = \int_k^\infty (u-k) \dfrac{1}{\sqrt{2\pi}} \exp\left(-u^2/2\right) du$.

4.3. The advanced (s,Q) inventory model

Janssen *et al*. (1998) analysed an (R, s, Q) inventory model, which was shown to be especially suitable for slow moving items such as spare parts. As in the MARS case we have the option to order every day, the (R, s, Q) model reduces to an (s, Q) model which is a special case, which in addition can be combined with the forementioned intermittent forecasting procedure. This inventory model is based on some important assumptions, some of which are based on the MARS situation ((a), (c), (e)):

(a) lead-time will be considered as fixed and integer;
(b) both the demand during lead-time (if positive) and the undershoot of the reorder level are distributed as mixed Erlang distributions;
(c) the reorder point s is positive;
(d) daily demand is i.i.d. and considered as a continuous variable;
(e) a service criterion (the so-called fill rate performance) is used with back ordering possibility;
(f) the order quantity Q is fixed a priori.

Dunsmuir and Snyder (1989) developed an (s, Q) inventory model where intermittent demand was modelled as a compound Bernoulli process (CBM), that is, with a fixed probability a positive demand during a time unit occurs, otherwise there is zero demand. Janssen *et al.* have adapted the method presented by Dunsmuir and Snyder, taking into account undershoot of the reorder point, possible shortages at the beginning of a replenishment cycle and the possibility of stochastic lead-times and a periodic review period. Using their method of approach the fill rate service equation (also known as P_2-service equation) for the (s, Q) model, can be shown to look as:

(14) $$1 - P_2 = \left\{ p_L (E(Z^* - s)^+ - E(Z^* - s - Q)^+) + (1 - p_L)(E(U - s)^+ - E(U - s - Q)^+) \right\} / Q$$

from which the reorder level *s* can be determined when *Q* is fixed. Here and in the sequel we employ the same notation as in the previous section. However, we omit the indices.

Management wanted the order quantity to be at least equal to the demand during lead-time. As $E(Z^*(L)) = E(Z(L)) / p_L$ and $p_L = 1 - (1 - p)^L$, we decided in favour of $\hat{Z}^*(L) = \hat{Z}(L) / p_L$ in the *Q* fixing, as it is in general more conservative than $\hat{Z}(L)$. Thus the order quantity *Q* was fixed as follows:

(15) $$\begin{aligned} Q &= EOQ \quad if \quad EOQ > 1.5 \hat{Z}^*(L) \\ &= 1.5 \hat{Z}^*(L), \quad elsewhere \end{aligned}$$

Besides, Janssen *et al.* (1998) have shown that when a more sophisticated approach is followed by minimizing the ordering cost plus the holding cost subject to the fill rate constraint, the *EOQ* is near to optimality when the holding cost is small. So a more complex simultaneous procedure for fixing *s* and *Q* seems not appropriate.

To solve the above service equation we evaluate expressions of the type $E(\cdot)^+$ using mixed Erlang distributions (cf. Janssen *et al.*, 1998; Tijms and Groenevelt, 1984), which are fitted on the basis of the forecasted moments of their corresponding stochastic variables *U* and $Z^* = Z^*(L) + U$. The first two moments of the undershoot *U* can be approximated in the following way, using renewal theory:

(16) $$E(U) \approx E(D^2) / (2E(D)) = E\left(D^{*2}\right) / (2E(D^*))$$

(17) $$E(U^2) \approx E(D^3) / (3E(D)) = E(D^{*3}) / (3E(D^*))$$

The variance of *U* follows from $Var(U) = E(U^2) - (E(U))^2$. From experience we observe right skewness in the D^* process. If we assume that D^* can be approximated by a gamma distribution, then

(18) $$E(D^{*3}) = (1 + c_{D^*}^2)(1 + 2c_{D^*}^2) a^3$$

In this way D^* is only a function of its first two moments, which can be estimated by the forecasting equations (4), (10) and (11). Using (16) an estimate of $\hat{U}$ for $E(U)$ can be obtained:

$$E(U) \approx \frac{Var(D^*) + [E(D^*)]^2}{2E(D^*)} = \frac{\sigma^2 + a^2}{2a}, \quad \hat{U} = \frac{\hat{\sigma}^2 + \hat{a}^2}{2\hat{a}}$$

Analogously we obtain $V\hat{a}r(U)$ from (16) through (18).

As $Var(Z^*(L)) = Var(Z(L)) / p_L - (1 - p_L)[E(Z(L))]^2 / p_L^2$ (cf. Janssen *et al.*, 1998) we obtain:

$$V\hat{a}r(Z^*(L)) = V\hat{a}r(Z(L)) / \hat{p}_L - (1 - \hat{p}_L)[\hat{Z}(L)]^2 / \hat{p}_L^2$$

where $\hat{Z}(L) = \hat{a}\hat{p}L$ and $V\hat{a}r(Z(L)) = V\hat{a}r(E(L))$ (cf. (9)). At last, we need the relations $\hat{Z}^* = \hat{Z}^*(L) + \hat{U}$ and $V\hat{a}r(Z^*) = V\hat{a}r(Z^*(L)) + V\hat{a}r(U)$.

By renewal theory Janssen *et al.* (1998) have shown that the expected average physical stock is equal to

(19) $$\mu_1(s,Q) = p_L \frac{E\left(\left[s+Q-Z^*(L)\right]^+\right)^2 - E\left(\left[s-Z^*(L)\right]^+\right)^2}{2Q} + (1-p_L)\frac{(s+Q)^2+s^2}{2Q}$$

It should be noted that in all cases we analyzed, which had high P_2-service levels and positive reorder points, the complex formula above could safely be approximated by the following simple one:

(20) $$\mu_2(s,Q) = s + Q/2 - E\,Z(L)$$

5. Simulation experiments

Monte Carlo experimentation will be employed to study the performance of the combined simple and advanced forecasting/inventory control procedures in the previous sections. Using those regimes, we will investigate by simulation both the attained (P_2-)service and average inventory levels thereby simulating data which resemble the company's usage data for various spare parts (cf. Table 2), so that the results are relevant for the company. An important aspect of the simulation study is the use of the forecasting procedures to estimate the demand parameters. Janssen *et al.* (1998) tested the CBM method with <u>known</u> demand parameters. However, using estimated parameter values will influence the performance of the system (Silver and Rahnama, 1987; Strijbosch *et al.*, 1997; Strijbosch and Moors, 1999). This section proceeds as follows. Firstly, we describe the design of the simulation program. Next the results of the simulations will be presented and discussed.

5.1. Simulation design

The simulation program is set up according to the 'next event' principle (Kleijnen and van Groenendaal, 1994). This principle means that after taking care of all events on a certain day, the programs continues to the day of the following event. The events, which may occur, are:

- A delivery by the supplier, resulting in an increase of the inventory position.
- A usage of the spare part, leading to an update of the forecasts of mean time between demands, mean demand size, and corresponding mean absolute deviation of forecast error.
- The inventory position gets under the reorder level, which evokes an order.

Demand sizes are generated according to a mixed Erlang distribution, whereas the time between demand occurrences is generated mostly according to a geometric distributed variable +1 (the interarrival time is at least 1). Using a run-in period of 100 demands measurements of performance are obtained after 100,000 demand occurrences. After each demand occurrence the forecast parameters $\hat{p}, \hat{a}$ and $\hat{\sigma}$ are updated. However, after each period of 90 days the decision parameters s and Q are recalculated using the corresponding service equation, thereby substituting the present values of the forecast parameters. Next, (19) and (20) are evaluated and averaged at the end of the simulation run. As is common practice a small value for ω is chosen ($\omega = 0.025$).

The nature of demand may normally considered to be reasonable stationary. Replacing parts by better alternatives cause dramatic changes in the demand pattern, for example. These causes cannot be accounted for by forecasting. The events, which influence the demand pattern strongly, should be accounted for by the 'management-by-exception' principle. Thus, relatively small values for α and β are chosen ($\alpha = \beta = 0.05$). The most important assumptions that are being used throughout the various simulations are:

(a) non-integer values for the reorder point and the order size generated by the simulation program are being rounded upwards and

(b) demand sizes generated by the program are being rounded to integers with a minimum of one.

The main output of the simulation program is as follows:

(a) Average (physical) inventory level $\mu(s, Q)$ as a result of the simulation.

(b) Average inventory level according to formulae (19) and (20).

(c) Attained P_2-service level, SL, defined by

(21) $$SL = 1 - \frac{mean\ shortage\ per\ cycle}{mean\ demand\ per\ cycle},$$

Mean shortage per cycle is defined as the accumulated shortage at the end of all replenishment cycles during the simulation horizon divided by the number of cycles, and *mean demand per cycle* is defined by aggregated demand over the simulation horizon divided by the number of cycles. A correction for a possible shortage at the beginning of the replenishment cycle seems not necessary as the service levels used in the simulation and desired by the company are very high (0.95 and 0.99).

5.2. Simulation results

The simple inventory control procedure will be denoted by STM (standard method), the advanced one by CBM (compound Bernoulli method). Table 2 has been used to set constant and varying values for the parameters of the control procedures in the simulation. The simulation yields the performances (realized service level, and average inventory level as measured during the simulation) of both procedures for each of the two desired service levels (0.95 and 0.99) as a function of one varying key variable. The constant (varying) values of the parameters are set as follows:

$$E(A) = 25\ (5, 10, 15, 20, 25, 50, 75, 100, 200),$$

$E(D^*) = 3$,

$Var(D^*) = 9\ (0, 2, 4, 6, 8, 10, 15, 20)$,

$E(L) = 20\ (5, 10, 20, 30, 40, 50)$,

$c_A = \sqrt{1 - 1/(E(A))}\ (0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6)$.

When, as is the case in the underlying CBM, the interarrival time has a geometric distribution with minimal value 1, the corresponding coefficient of variation c_A equals $\sqrt{1 - 1/E(A)}$, which is close to 1 in many cases. As there is some doubt on the validity of the assumption of the geometric distribution, we have performed a simulation experiment where the interarrival time is generated by a gamma distribution with varying c_A.

Table 3: *Attained service levels for varying delivery times; tabulated are* $100 * (P_2 - \hat{P}_{2,\,CBM})$, *and* $100 * (P_2 - \hat{P}_{2,\,STM})$ *within parentheses.*

P_2	Delivery time											
	5		10		20		30		40		50	
0.95	2	(18)	2	(14)	2	(11)	2	(10)	2	(9)	2	(9)
0.99	1	(13)	1	(10)	1	(8)	1	(7)	1	(6)	1	(6)

Table 3 shows the simulation results for varying lead-time. As lead-time increases the performance of CBM is consistent. However, the attained service level is 1-2% under the desired one, which is mainly due to the fact that the distribution parameters are unknown and substituted by estimated values. A better approach of the desired service level could simply be obtained in practice by upgrading the P_2-service a little. On the contrary, STM yields a service level that is much lower in the first place and is not consistent. The increasing service level for increasing lead-times is due to the fact that the distribution of demand during lead-time will be closer to the normal distribution as lead-time increases. There is no obvious way to correct STM such that the attained service level would be closer to the desired level.

Table 4: *Attained service levels for varying mean interarrival times; tabulated are* $100 * (P_2 - \hat{P}_{2,\,CBM})$, *and* $100 * (P_2 - \hat{P}_{2,\,STM})$ *within parentheses.*

P_2	Mean interarrival time																	
	5		10		15		20		25		50		75		100		200	
0.95	2	(7)	2	(8)	2	(10)	2	(11)	2	(11)	2	(15)	2	(17)	2	(18)	3	(22)
0.99	1	(4)	1	(5)	1	(6)	1	(7)	1	(8)	1	(10)	1	(12)	1	(14)	1	(18)

Table 4 shows the results for varying mean interarrival times using an interarrival time which is geometric. Again, as should be, CBM produces a consistent level of the attained service. However, STM turns out to be extremely sensitive to $E(A)$.

Table 5 shows the results for varying variance of demand size. The decrease of the attained service level with CBM and increasing $Var(D^*)$ is mainly due to the fact that the factor 1.25 used in order to obtain an estimate of the forecast error standard deviation, should be larger (Jacobs and Wagner, 1989). With STM, again the normality assumption is less appropriate with increasing $Var(D^*)$, yielding a decreasing service level.

Table 5: *Attained service levels for varying variance of* D^**; tabulated are* $100*(P_2 - \hat{P}_{2,\,CBM})$, *and* $100*(P_2 - \hat{P}_{2,\,STM})$ *within parentheses.*

P_2	Variance of D^*															
	0		2		4		6		8		10		15		20	
0.95	1	(6)	1	(8)	1	(9)	1	(10)	2	(11)	2	(12)	3	(14)	4	(16)
0.99	0	(3)	0	(4)	0	(5)	1	(6)	1	(7)	1	(8)	1	(10)	2	(12)

Table 6: *Attained service levels for varying coefficient of variation of the interarrival time; tabulated are* $100*(P_2 - \hat{P}_{2,\,CBM})$, *and* $100*(P_2 - \hat{P}_{2,\,STM})$ *within parentheses.*

P_2	Coefficient of variation of interarrival time													
	0.4		0.6		0.8		1.0		1.2		1.4		1.6	
0.95	-2	(12)	-1	(14)	1	(16)	2	(18)	5	(21)	7	(23)	10	(25)
0.99	0	(10)	0	(11)	0	(12)	1	(14)	2	(16)	3	(18)	5	(20)

Table 6 shows the extent of robustness of CBM against violating the assumption that interarrival times are geometrically distributed. For these simulated situations the attained service levels diminish, which is mainly due to the fact that A_z and D_z are not i.i.d. (as is used for both STM and CBM, cf. (7) and (8)) when c_A deviates from the corresponding value of a geometric variable (Janssen *et al.*, 1998).

For all different simulation situations (62 in number) $\mu_1(s,Q)$, $\mu_2(s,Q)$ and $\hat{\mu}(s,Q)$ (the average physical inventory during the simulation run) are evaluated. We found that $0\% < \left(\dfrac{\mu_1(s,Q) - \mu_2(s,Q)}{\hat{\mu}(s,Q)}\right) < 6\%$,

$-4\% < \dfrac{\mu_2(s,Q)-\hat{\mu}(s,Q)}{\hat{\mu}(s,Q)} < 7\%$ and $1\% < \left(\dfrac{\mu_1(s,Q)-\hat{\mu}(s,Q)}{\hat{\mu}(s,Q)}\right) < 12\%$ [1]. These figures show that the preference for formula (20) is appropriate. It turns out that formula (20) is a very useful formula to obtain an estimate of the average inventory in the described situations.

6. CONCLUSIONS

This paper tried to give the management of MARS an idea whether going from an intuitive control to a more consistent complex combined forecasting/replenishment approach, would pay off. The results of the new approach were such that they decided to implement it in the near future.

Literature

Brown R.G. (1963). *Smoothing, forecasting and prediction.* Prentice-Hall: Englewood Cliffs, N.J.

Croston J.D. (1972). Forecasting and stock control for intermittent demands. *Operational Research Quarterly* **23**, 289-303.

Dunsmuir, W.T.M., and R.D. Snyder (1989). Control of inventories with intermittent demand. *European Journal of Operational Research* **40**, 16-21.

Jacobs R.A. and H.M. Wagner (1989). Reducing inventory system costs by using robust demand estimators *Mgmt Sci* **35**, 771-787.

Janssen F.B.S.L.P., Heuts R.M.J. and A.G. de Kok (1998). On the (R,s,Q) inventory model when demand is modelled as a compound Bernoulli process. *European Journal of Operational Research* **104**, 423-436.

Johnston F.R. and J.E. Boylan (1996). Forecasting for items with intermittent demand. *Journal of the Operational Research Society* **47**, 113-121.

Kleijnen J.P.C. and W. van Groenendaal (1994). *Simulation, a statistical perspective.* Wiley: Chichester.

Mood A.M., Graybill F.A. and D.C. Boes (1974). *Introduction to the theory of statistics.* Third Edn. McGraw-Hill: Tokyo.

van der Schoot, E.H.M. (1998). *Voorraadbeheersing van reserveonderdelen bij MARS* (in Dutch). Masters Thesis, Tilburg University, The Netherlands.

Silver E.A., Pyke D. and R. Peterson (1998). *Inventory management and production planning and scheduling*. Third Edn. John Wiley: New York.

[1] The largest percentages occur with the lowest average inventory levels.

Silver E.A. and M.R. Rahnama (1987). Biased selection of the inventory reorder point when demand parameters are statistically estimated. *Engineering Costs and Production Economics* **12**, 283-292.

Strijbosch L.W.G., Moors, J.J.A. and A.G. de Kok (1997). On the interaction between forecasting and inventory control. *Research Memorandum FEW* **742**, Tilburg University: The Netherlands.

Strijbosch L.W.G. and J.J.A. Moors (1999). Inventory control: the impact of unknown demand distribution. *Research Memorandum FEW* **770**, Tilburg University: The Netherlands.

Tijms H.C. and H. Groenevelt (1984). Simple approximations for the reorder point in periodic and continuous review (s, S) inventory systems with service level constraints. *European Journal of Operational Research* **17**, 175-190.

Willemain T.R., Smart C.N., Shockor J.H. and P.A. DeSautels (1994). Forecasting intermittent demand in manufacturing: a comparitive evaluation of Croston's method. *International Journal of Forecasting* **10**, 529-538.